Exercises 2

Exercise 2.1

Radiation illuminating a sodium surface causes an electron to be emitted at a speed of 100 m/s. What is the wavelength of the incident beam?

(ionization energy $E_I = 2.28$ eV et 1 eV = $1.602 \cdot 10^{-19}$ J)

We assume that the incident beam transmits all its energy to overcome the ionization energy and give the electron a velocity, i.e.

$$E_{photon} = E_{kin} + EI$$

By expanding , then replacing ν in the equation, we find:

By developing $E_{photon} = h\nu$ and $E_{kin} = \frac{mv^2}{2}$, then replacing ν in the equation $\lambda = \frac{c}{\nu}$ we find:

$$\lambda = \frac{hc}{\frac{mv^2}{2} + EI}$$

As 1 eV~= $1.6 \cdot 10^{-19}$ J, we obtain :

$$\frac{6.626 \cdot 10^{-34} \cdot 3 \cdot 10^{8}}{\frac{9.109 \cdot 10^{-31} \cdot 100^{2}}{2} + 2.28 \cdot 1.6 \cdot 10^{-19}} \text{m} = 5.44 \cdot 10^{-7} \text{m} = 544 \text{ nm}$$

It is therefore a green ray of light.

Exercise 2.2

A lamp emits energy equivalent to 7 J every second. In 10 s, $9.4 \cdot 10^{19}$ photons are emitted. Assuming that all these photons have the same frequency, what is this frequency in PHz?

Using the formula E = hν, we calculate for a photon :

$$\nu = \frac{E}{h} \cong \frac{\frac{7 \cdot 10}{9.4 \cdot 10^{19}}}{6.626 \cdot 10^{-34}} \cong 1 \cdot 10^{15} \text{ Hz}$$

The frequency is therefore 1 PHz.

Exercise 2.3

What is the speed (in m/s) of a neutron with a wavelength of $4.43 \cdot 10^{-1}$ nm? Knowing that the size of objects detected by a measurement depends on the wavelength used, give a possible application for such short wavelengths.

Using de Broglie's relation :

$$p = mv = \frac{h}{\lambda}$$

we obtain :

$$v = \frac{h}{\lambda \cdot m_n} = \frac{6.626 \cdot 10^{-34}}{4.43 \cdot 10^{-10} \cdot 1.675 \cdot 10^{-27}} = 893 \text{ m/s}$$

Such short wavelengths can be used to detect the position of atoms within molecules. One such technique is neutron diffraction, which can be used to obtain precise molecular structures.

Exercise 2.4

A photon with a wavelength of 150 pm ejects an electron from an atom with an ionization energy of $1.12 \cdot 10^{-15}$ J. How fast will this electron be emitted?

We use the same equations as for Exercise 2.1:

$$v = \sqrt{\frac{2(h\nu - EI)}{m_E}} = \sqrt{\frac{2\left(h \cdot \frac{c}{\lambda} - EI\right)}{m_E}} = \sqrt{\frac{2\left(6.626 \cdot 10^{-34} \cdot \frac{3 \cdot 10^{8}}{150 \cdot 10^{-12}} - 1.12 \cdot 10^{-15}\right)}{9.109 \cdot 10^{-31}}}$$

$$= 2.12 \cdot 10^{7} \text{ m/s}$$

Exercise 2.5

The energy required to ionize a given atom is $3.44 \cdot 10^{-18}$ J. This atom absorbs a photon by emitting an electron at $1.03 \cdot 10^{6}$ m/s.

What is the wavelength and type (UV, visible, infrared, gamma, ...) of the photon that was absorbed?

Again, using the equations from Exercise 2.1, we obtain :

$$\lambda = \frac{hc}{\frac{1}{2} m_e v^2 + EI} \cong \frac{6.626 \times 10^{-34} \cdot 2 \times 10^{8}}{0.5 \cdot 9.109 \times 10^{-31} \cdot (1.03 \times 10^{6})^2 + 3.44 \times 10^{-18}} \cong 5.07 \times 10^{-8} \text{m}$$

With a wavelength of 50.7 nm, this is UV radiation.

Exercise 2.6

What is the change in energy of a lithium atom emitting a photon with a wavelength of 683 nm? What is the color of the emitted photon?

$$E = \frac{hc}{\lambda} \cong 2.91 \times 10^{-19} \text{J}$$

The wavelength of 683 nm corresponds to red light.

Exercise 2.7

An electron is enclosed in a space whose size is of the order of magnitude of an atom: 100 pm. What is the minimum uncertainty of its momentum?

According to Heisenberg's inequality:

$$\Delta p \geq \frac{\hbar}{2\pi\Delta x} = \frac{h}{4\pi \cdot 10^{-10}} \cong 5.28 \times 10^{-25} \mathrm{kg} \cdot \mathrm{m/s}$$